Bond’s duration

Bond’s duration

It is a measure of the risk of a price change, which makes it possible to compare bonds of different maturities and different interest rates. The higher the duration value, the greater the risk of a bond.

Calculation of duration if interest is paid once a year

$$D = \frac{\sum_{t=1}^{n}\left(t \times \frac{C_t}{(1+YTM)^t}\right)}{P}$$

D – bond’s duration
P – bond price
C_t – cash flow from bond
YTM – yield to maturity
t – the period in which the payment of interest takes place

Calculation of duration if interest is paid more than once a year

$$D = \frac{\sum_{t=1}^{nm}\left(t \times \frac{C_t}{(1+\frac{YTM}{m})^t}\right)}{P} \div m$$

D – bond's duration
P – bond price
C_t – cash flow from bond
YTM – yield to maturity
t – the period in which the payment of interest takes place
m – number of interest payments during the year

The formula for calculating the approximate percentage change of the price of bonds when the rate of return changes on time to maturity

$$\frac{P_1 - P_0}{P_0} = -D \times \frac{(1+YTM_1)-(1+YTM_0)}{1+YTM_0}$$

P_0 – the price of bond before the change of rate of return
P_1 – the price of bond after the change of rate of return
D – bond's duration
YTM_0 – yield to maturity before the change
YTM_1 – yield to maturity after the change

Duration of the bond portfolio

$$D_p = \sum_{i=1}^{n}(w_i \times D_i)$$

D_p – duration of the bond portfolio
w_i – the weight of individual bonds in the portfolio
D_i – duration of individual bonds in the portfolio

Modified duration

It determines the approximate percentage change in the price of a bond due to a change in the rate of return by 1 percentage point

Modified duration if interest is paid once a year

$$MD = \frac{D}{1 + YTM}$$

MD – modified duration
D – duration
YTM – yield to maturity

Modified duration if interest is paid more than once a year

$$MD = \frac{D}{1 + \frac{YTM}{m}}$$

MD – modified duration
D – duration
YTM – yield to maturity
m – number of interest payments during the year

The formula for calculating the approximate percentage change of the price of bonds when the rate of return changes on time to maturity

$$\frac{P_1 - P_0}{P_0} = -MD \times (YTM_1 - YTM_0)$$

P_0 – the price of bond before the change of rate of return
P_1 – the price of bond after the change of rate of return
MD – modified duration
YTM_0 – yield to maturity before the change
YTM_1 – yield to maturity after the change

Task 1

1. Prepare a table of cash flows from bonds.
2. Estimate the discount factor which we need to calculate the discounted cash flow. Use the following formula:

$$Discount\ factor = \frac{1}{\left(1 + \frac{YTM}{m}\right)^t}$$

3. Multiply the cash flow by the discount factor and calculate the discounted cash flow

Task 1

4. Sum up all discounted cash flows and calculate bond price (P)
5. Multiply the discounted cash flow by the (t) period
6. Sum up all t×PV_Ct
7. Use the following formula to estimate the duration:

$$D = \frac{\sum_{t=1}^{n}\left(t \times \frac{C_t}{(1+YTM)^t}\right)}{P}$$