4. Estimating the yield to maturity of coupon bonds

Definition - yield to maturity (YTM)

It is the annual income rate of the investor who buys the bonds and processes them until maturity

Calculation of yield to maturity

$$P = \sum_{t=1}^{n} \frac{C_t}{(1+YTM)^t}$$

P – bond price
C_t – cash flow from bond
YTM – yield to maturity (an unknown)
t – the period in which the payment of interest takes place

When interest is paid more than once a year

$$P = \sum_{t=1}^{n} \frac{C_t}{(1+\frac{YTM}{m})^t}$$

P – bond price
C_t – cash flow from bond
YTM – yield to maturity (an unknown)
t – the period in which the payment of interest takes place
m – number of interest payments during the year

Trial and error calculation

In the trial and error method, to determine the value of YTM, cash flows should be discounted with various discount rates (which we assume) until the discounted cash flows will be equal to the price of the bonds.

Trial and error calculation

$$YTM = r_1 + \frac{NPV_1 \times (r_2 - r_1)}{NPV_1 + |NPV_2|}$$

YTM – yield to maturity
r1 – discount rate for NPV1 (positive)
r2 – discount rate for NPV2 (negative)
NPV1 – NPV positive for investments in bonds (close to 0)
NPV2 – NPV negative for investments in bonds (close to 0)

Trial and error calculation

1. Prepare a table of cash flows from bonds
2. Assume any level of the discount rate and discount cash flows
3. If the NPV is positive, then we increase the discount rate and re-discount the flows to get also the negative NPV
4. When the NPV is negative, we decrease the discount rate and also re-discount the flows.
5. After calculating the positive and negative NPV as close as possible to 0, we use the formula for the calculation of YTM

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5. Estimating return on the bond portfolio and realized compound yield

Return on the bond portfolio

When dealing with a portfolio of bonds, the rate of return (YTM) from it is not simply the arithmetic mean rate of return estimated for individual bonds in the portfolio. It is calculated by determining the cash flows from all the bonds (added) and determining the interest rate for which the present value of cash flows will be equal to the market value of the portfolio*.

* N.B. you will deal with big numbers AND you can and should use the formula from the last meeting

Realized compound yield

Yield to maturity assumes that interest on bonds is reinvested at a rate equal to its size. Only in this case the investor can achieve a return rate equal to the yield to maturity. If the investor assumes that the reinvestment rate is different from the yield to maturity, the return on investment will also differ from it. In this case, you can calculate the realized compound yield.

Procedur for calculating the realized compound yield

1. To assume the reinvestment rate and to estimate the value of future interest at the end of the investment period, use the formula on the future value of annuity:

$$A_n = I \times \left[\frac{(1 + r_{re})^n - 1}{r_{re}} \right]$$

A_n – future value of annuity
I – the value of coupon interest
r_{re} – reinvestment rate
n – total number of periods of payments

Procedur for calculating the realized compound yield

2. Estimate the face value of the bond or the price at the end of the investment period.
3. Sum up values calculated in points 1 and 2 to get total positive cash flows.
4. In the last stage, we use the formula:

$$RCY = \left(\frac{FCF}{P} \right)^{\frac{1}{n}} - 1$$

RCY – realized compound yield
FCF – future value of total positive cash flows from bonds
P – bond price
n – total number of periods of payments